Fractions I :

1) prime factorization
2) multiplying fractions
3) dividing fractions
4) areas of triangles
5) solving basic fraction
    equations

prime factorization

Example 1 Find the prime factorization of 150.

$$2 | 150$$
$$3 | 75$$
$$5 | 25$$
$$5$$

$150 = \boxed{2 \cdot 3 \cdot 5^2}$

use only prime numbers to break it down.

Example 2 | 1200

$$2 | 1200$$
$$2 | 600$$
$$2 | 300$$
$$2 | 150$$
$$3 | 75$$
$$5 | 25$$
$$5$$

$1200 = \boxed{2^4 \cdot 3 \cdot 5}$

Exercises Find the prime factorization of:

1) 1450

2) 480

3) 575

$\boxed{\text{Simplifying Fractions}}$ ("Reducing")

your goal: the numerator and denominator have no common factors

$\boxed{\text{Example 1}}$ Simplify $\dfrac{30x}{36x}$

$\begin{array}{r} 2\,|\,30 \\ 3\,|\,15 \\ 5 \end{array}$ $\begin{array}{r} 2\,|\,36 \\ 2\,|\,18 \\ 3\,|\,9 \\ 3 \end{array}$

$$\frac{30x}{36x} = \frac{\cancel{2}\cdot\cancel{3}\cdot 5\cdot\cancel{x}}{\cancel{2}\cdot 2\cdot\cancel{3}\cdot 3\cdot\cancel{x}} = \boxed{\frac{5}{6}}$$

$\boxed{\text{Example 2}}$ Simplify $\dfrac{45x^2y}{27xy^3}$

$\begin{array}{r} 3\,|\,45 \\ 3\,|\,15 \\ 5 \end{array}$ $\begin{array}{r} 3\,|\,27 \\ 3\,|\,9 \\ 3 \end{array}$

$$\frac{45x^2y}{27xy^3} = \frac{\cancel{3}\cdot\cancel{3}\cdot 5\cdot x\cdot\cancel{x}\cdot\cancel{y}}{\cancel{3}\cdot\cancel{3}\cdot 3\cdot\cancel{x}\cdot y\cdot y\cdot\cancel{y}} = \boxed{\frac{5x}{3y^2}}$$

$\boxed{\text{Practice Exercises}}$ Simplify

① $\dfrac{12}{28}$

② $\dfrac{15}{27}$

③ $\dfrac{25x}{75x^2}$

④ $\dfrac{36y^3}{72y}$

⑤ $\dfrac{18xyz}{23xy}$

⑥ $\dfrac{42ab^2c}{30abc^3}$

multiplying fractions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \qquad \text{where} \quad b \neq 0, d \neq 0$$

cancel out common factors when you can.

Example 1
$$-\frac{6}{7} \cdot \frac{5}{12} = -\frac{\cancel{6}^{1}}{7} \cdot \frac{5}{\cancel{12}_{2}} = \boxed{\frac{-5}{14}}$$

Example 2
$$\frac{27y^3}{21} \cdot \frac{7}{18y^2} = \frac{27 y^3}{21} \cdot \frac{7}{18y^2} = \boxed{\frac{y}{2}}$$

Example 3
$$\left(\frac{-5}{12}\right)^2 = \left(\frac{-5}{12}\right)\left(\frac{-5}{12}\right) = \boxed{\frac{25}{144}}$$

practice problems

① $\dfrac{3}{5} \cdot \dfrac{1}{2}$

② $\dfrac{7}{8x} \cdot \dfrac{-2}{3}$

③ $-\dfrac{24x}{5} \cdot \dfrac{-15}{8x^3}$

④ $\left(\dfrac{-2}{3}\right)^3$

⑤ $\dfrac{ac}{b} \cdot \dfrac{b^2}{a^3 c}$

⑥ $\dfrac{2x}{15x^2 y} \cdot \dfrac{3xy}{4z}$

Division of Fractions

definition: reciprocal

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

To divide fractions, multiply by the reciprocal.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \qquad \begin{array}{l} b \neq 0 \\ c \neq 0 \\ d \neq 0 \end{array}$$

Example 1 divide: $\frac{21a}{4} \div \frac{7a}{5}$

$$\frac{21a}{4} \div \frac{7a}{5} = \frac{21a}{4} \cdot \frac{5}{7a} = \frac{\overset{3}{\cancel{21a}}}{4} \cdot \frac{5}{\cancel{7a}} = \boxed{\frac{15}{4}}$$

Example 2 simplify $\dfrac{\frac{3y}{7}}{\frac{11}{y}}$ $\frac{3y}{7} \cdot \frac{y}{11} = \boxed{\frac{3y^2}{77}}$

practice exercises divide:

① $-\frac{3}{4} \div \frac{3}{8}$

② $\frac{18x}{5} \div \frac{2}{5x}$

③ $-\frac{9}{2} \div \frac{-1}{3}$

④ $-\frac{5}{3} \div 2y$

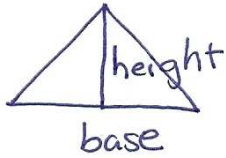⑤ $\frac{5x^2}{y} \div \frac{10x^3}{y^3}$

⑥ $\dfrac{\frac{2x}{5}}{\frac{7}{10}}$

⑦ $\dfrac{\frac{5x}{3y}}{\frac{15x^2}{6y}}$

Areas of Triangles and Rectangles
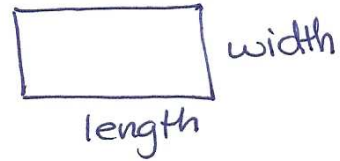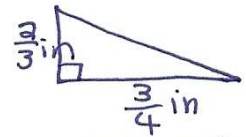
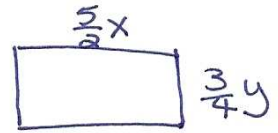Area of triangle $= \frac{1}{2}bh$

Area of Rectangle $= L \cdot W$

base, height (triangle diagram)

width, length (rectangle diagram)

Example 1 Find the area of the triangle

$\frac{2}{3}$ in, $\frac{3}{4}$ in

$\frac{1}{2}$ b h

$A = \frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) = \frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) = \boxed{\frac{1}{4} \text{ in}^2}$

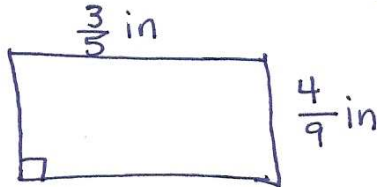**Example 2** Find the area of the rectangle

$\frac{5}{2}x$, $\frac{3}{4}y$

$A = L \cdot W = \left(\frac{5}{2}x\right)\left(\frac{3}{4}y\right) = \boxed{\frac{15}{8}xy}$
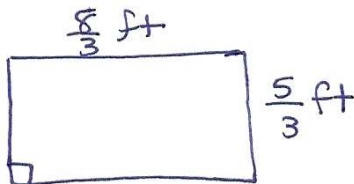
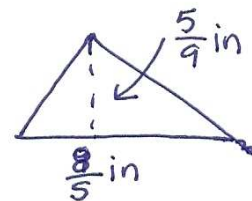**practice exercises** Find the area of each figure

① Rectangle — $\frac{3}{5}$ in, $\frac{4}{9}$ in

② Rectangle — $\frac{8}{3}$ ft, $\frac{5}{3}$ ft

③ triangle — $\frac{1}{3}$ in, $\frac{5}{8}$ in

④ triangle — $\frac{5}{9}$ in, $\frac{8}{5}$ in
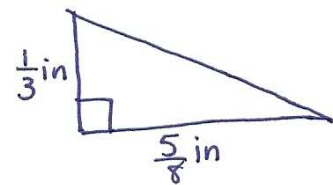
Solving Equations with Fractions

SECTION ⬤ SOLVING EQUATIONS CONTAINING FRACTIONS (CONTINUED)

To SOLVE AN EQUATION IN x

Step 1. If fractions are present, multiply both sides of the equation by the LCD of the fractions.

Step 2. If parentheses are present, use the distributive property.

Step 3. Combine any like terms on each side of the equation.

Step 4. Use the addition property of equality to rewrite the equation so that variable terms are on one side of the equation and constant terms are on the other side.

Step 5. Divide both sides by the numerical coefficient of x to solve.

Step 6. Check the answer in the *original equation*.

Solve: $\dfrac{x}{15} + 2 = \dfrac{7}{3}$

$$15\left(\dfrac{x}{15} + 2\right) = 15\left(\dfrac{7}{3}\right) \quad \text{Multiply by the LCD 15.}$$

$$15\left(\dfrac{x}{15}\right) + 15 \cdot 2 = 15\left(\dfrac{7}{3}\right)$$

$$x + 30 = 35$$

$$x + 30 + (-30) = 35 + (-30)$$

$$x = 5$$

Check to see that 5 is the solution.

Example 1   Solve $-\dfrac{3}{5}x = 6$

$(5) \cdot -\dfrac{3}{5}x = 6 \,(5)$

$-3x = 30$

$\dfrac{-3x}{-3} = \dfrac{30}{-3}$

$\boxed{x = -10}$

Example 2   Solve $\dfrac{1}{6} + \dfrac{x}{4} = \dfrac{17}{12}$

$lcd = 12$

$\overset{2}{\cancel{12}}\left(\dfrac{1}{\cancel{6}}\right) + \overset{3}{\cancel{12}}\left(\dfrac{x}{\cancel{4}}\right) = \cancel{12}\left(\dfrac{17}{\cancel{12}}\right)$

$\begin{array}{r} 2 + 3x = 17 \\ -2 \qquad\quad -2 \end{array}$

$3x = 15$

$\boxed{x = 5}$

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practice problems   SOLVE:

① $\dfrac{2}{9}y = -\dfrac{4}{3}$

② $\dfrac{x}{7} - 3 = -\dfrac{6}{7}$

③ $\dfrac{y}{5} + 2 = \dfrac{11}{5}$

④ $\dfrac{x}{5} - \dfrac{5}{4} = \dfrac{x}{2} - \dfrac{1}{20}$

⑤ $\dfrac{3}{4}x = \dfrac{5}{8}$

⑥ $-\dfrac{2}{3}x = \dfrac{-4}{7}$